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# SM3H Test Review 5.1-5.7 Key

**2017-18**

**Name** ______________________ **Date** ________________ **Period**________

**Evaluate the logarithm without a calculator. Show work!**

1. $\log_6\left(\frac{1}{36}\right) = \boxed{-2}$
   $\frac{1}{36} = 6^{-2}$

2. $\log_8(32) = \boxed{5/3}$
   $8^x = 32$
   $2^{3x} = 2^5$
   $3x = 5$, $x = 5/3$

3. $\log 0.0001 = \boxed{-4}$
   $.0001 = 10^{-4}$

4. $\log_{21}\sqrt{21} = \boxed{1/2}$
   $\sqrt{21} = (21)^{1/2}$

5. $\ln\frac{1}{\sqrt{e^{11}}} = \boxed{-11/2}$
   $\frac{1}{\sqrt{e^{11}}} = e^{-11/2}$

6. $\log_7 343 = \boxed{3}$
   $343 = 7^3$

7. $\log_6 6^2 = \boxed{2}$

8. $e^{\ln 20} = \boxed{20}$

9. $\log_8\frac{1}{64} = \boxed{-2}$
   $\frac{1}{64} = \frac{1}{8^2} = 8^{-2}$

10. $\ln e = \boxed{1}$

11. $\log_{12} 1 = \boxed{0}$

Find the following using a calculator. Round to the nearest ten thousandths. (4-decimals)

12. $\log 32$
    $1.5051$

13. $\ln 0.98$
    $-.0202$

14. $\log(-3)$
    undefined

15. $5^{3.2}$
    $172.4662$

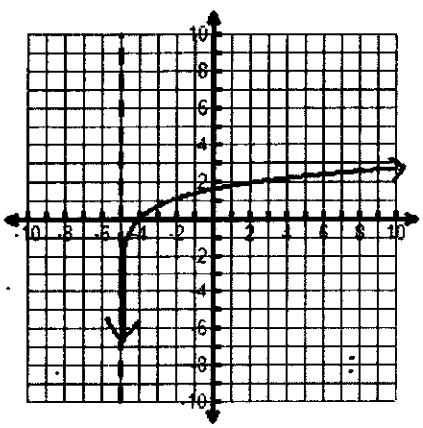
Solve the equation by changing it to exponential form. Round to the nearest ten thousandths.

16. $\log_4 x = \frac{1}{2}$
    $4^{1/2} = x$
    $\boxed{2 = x}$

17. $\log x = -4$
    $10^{-4} = x$
    $\boxed{.0001 = x}$

18. $\ln x = 2$
    $e^2 = x$
    $\boxed{7.3891}$

19. Determine the function that best describes the given graph.




a. $y = \ln x - 5$

b. $y = \ln(x - 5)$

c. $y = \ln x + 5$

d. $y = \ln(x + 5)$
   shift left 5

20. Describe how to transform the graph of the basic function $g(x)$ into the graph of the given function $f(x)$.

$g(x) = \ln x$; $f(x) = \ln(-x) - 7$ Reflection across y-axis, down 7.

Rewrite the expression as a sum or difference or multiple of logarithms.

21. $\log_2(5\sqrt[3]{12})$

$\log_2 5 + \log_2 \sqrt[3]{12}$

$\boxed{\log_2 5 + \tfrac{1}{3}\log_2 12}$

22. $\log_8\left(\frac{2x-3}{x^4}\right) = \log_8(2x-3) - \log_8 x^4$

$= \boxed{\log_8(2x-3) - 4\log_8 x}$

Use the product, quotient and power rules of logarithms to rewrite the expression as a single logarithm. Assume that all variables represent positive real numbers.

23. $\log_3 6 - \log_3 a$

$\log_3\left(\frac{6}{a}\right)$

24. $4\log x + 2\log y$

$\log x^4 + \log y^2$

$\boxed{\log(x^4y^2)}$

25. $2\log_4 3 + \frac{1}{2}\log_4(x-5) - \frac{1}{3}\log_4 x$

$\log_4 3^2 + \log_4\sqrt{x-5} - \log_4\sqrt[3]{x}$

$\boxed{\log_4\left(\frac{9\sqrt{x-5}}{\sqrt[3]{x}}\right)}$

Write the change of base rule to find the logarithm to the nearest ten thousandths.

26. $\log_{3.4} 210 = \frac{\log 210}{\log 3.4}$ or $\frac{\ln 210}{\ln 3.4}$ $= \boxed{4.3694}$

Write the expression in change of base using only the indicated logarithms.

27. $\log_4(x+y)$, use common logarithms

$\boxed{\frac{\log(x+y)}{\log 4}}$

28. $\log_2 13$, use natural logarithms

$\boxed{\frac{\ln 13}{\ln 2}}$

Find the exact solutions to the equation. Show work.

29. $\log_4(x-2) = -1$

$4^{-1} = x-2$

$\frac{1}{4} = x-2$

$\boxed{x = 2\tfrac{1}{4}}$

30. $3^{7x} = 243$

$3^{7x} = 3^5$

$7x = 5$

$\boxed{x = 5/7}$

Solve each equation. Show work. Round to the nearest thousandths if necessary.

31. $\log_4(x+5) = 3$

$x > -5$

$4^3 = x+5$

$64 = x+5$

$\boxed{59 = x}$

32. $\log_3(x+4) - \log_3 4 = \log_3 22$

$x > -4$

$\log_3\left(\frac{x+4}{4}\right) = \log_3 22$

$\frac{x+4}{4} = 22$

$x+4 = 88$

$\boxed{x = 84}$

$x > \frac{4}{3}$

33. $\log_5 4 + \log_5(3x-4) = 2$

$\log_5[4(3x-4)] = 2$

$5^2 = 4(3x-4)$

$25 = 12x - 16$

$41 = 12x$

$\boxed{x = \frac{41}{12}}$

34. $3e^{(2x-7)} = 8$

$e^{2x-7} = \frac{8}{3}$

$\ln(\frac{8}{3}) = 2x - 7$

$\frac{\ln(\frac{8}{3}) + 7}{2} = \frac{2x}{2}$

$x = \frac{\ln(\frac{8}{3}) + 7}{2} \approx \boxed{3.9904}$

35. $\log\left(\frac{3}{5}x - 2\right) = 5$  $x > \frac{10}{3}$

$10^5 = \frac{3}{5}x - 2$

$100{,}000 = \frac{3}{5}x - 2$

$100{,}002 = \frac{3}{5}x$

$\frac{5}{3}(100{,}002) = x$

$\boxed{x = 166{,}670}$

36. $4^{(x-5)} + 4 = 9$ $-4$ $-4$

$4^{x-5} = 5$

$\log 4^{(x-5)} = \log 5$

$(x-5)\log 4 = \log 5$

$x\log 4 - 5\log 4 = \log 5$

$x\log 4 = \log 5 + 5\log 4$

$x = \frac{\log 5 + 5\log 4}{\log 4} = \boxed{6.1610}$

$x > 1$ ($x > 5/2$)

37. $\log_3(x-1) - \log_3(2x-5) = 0$

$\log_3\left(\frac{x-1}{2x-5}\right) = 0$

$3^0 = \frac{x-1}{2x-5}$

$1 = \frac{x-1}{2x-5}$

$2x - 5 = x - 1$

$-x + 5 \quad -x + 5$

$\boxed{x = 4}$

38. $\log_2(x^2 - 2x) = 3$ $x < 0$ or $x > 2$

$2^3 = x^2 - 2x$

$8 = x^2 - 2x$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$\boxed{x = 4 \quad x = -2}$ (Check solutions) (find domain)

Find the inverse of each function. Show work.

39. $f(x) = \log(x+7) - 2$

$x = \log(y+7) - 2$

$x + 2 = \log(y+7)$

$10^{(x+2)} = y + 7$

$\boxed{10^{(x+2)} - 7 = f^{-1}(x)}$

40. $f(x) = 2\ln(8-x) + 5$

$x = 2\ln(8-y) + 5$

$x - 5 = 2\ln(8-y)$

$\frac{x-5}{2} = \ln(8-y)$

$e^{(\frac{x-5}{2})} = 8 - y$

$e^{(\frac{x-5}{2})} - 8 = -y$

$\boxed{f^{-1}(x) = 8 - e^{(\frac{x-5}{2})}}$

41. $f(x) = \log_3(3x-4) + 1$

$x = \log_3(3y-4) + 1$

$x - 1 = \log_3(3y-4)$

$3^{(x-1)} = 3y - 4$

$3^{(x-1)} + 4 = 3y$

$\boxed{\frac{3^{(x-1)} + 4}{3} = f^{-1}(x)}$

42. $f(x) = 5^{x-3} + 2$

$x = 5^{y-3} + 2$

$x - 2 = 5^{y-3}$

$\log_5(x-2) = y - 3$

$\boxed{\log_5(x-2) + 3 = f^{-1}(x)}$

43. $f(x)=e^{4x-5}-7$

$x = e^{4y-5} - 7$

$x+7 = e^{4y-5}$

$\ln(x+7) = 4y-5$

$\ln(x+7)+5 = 4y$

$\boxed{\dfrac{\ln(x+7)+5}{4} = f^{-1}(x)}$

44. $f(x)=-2\cdot 3^{5-2x}+1$

$x = -2\cdot 3^{5-2y} + 1$

$x-1 = -2\cdot 3^{5-2y}$

$\dfrac{x-1}{-2} = 3^{5-2y}$

$\log_3\left(\dfrac{x-1}{-2}\right) = 5-2y$

$\log_3\left(\dfrac{x-1}{-2}\right) - 5 = -2y$

$\boxed{f^{-1}(x) = \dfrac{\log_3\left(\frac{x-1}{-2}\right) - 5}{-2}}$

45. $f(x) = \frac{2x+3}{5x-2}$ Write the domain of the $f(x)$. Find $f^{-1}(x)$ and the domain of $f^{-1}(x)$.

$\boxed{\text{Domain of } f:\ x \neq \frac{2}{5}}$ $\boxed{\text{Domain of } f^{-1}:\ x \neq 2/5}$

$x = \dfrac{2y+3}{5y-2}$

$x(5y-2) = 2y+3$

$5xy - 2x = 2y+3$

$5xy - 2y = 2x+3$

$y(5x-2) = 2x+3$

$\boxed{f^{-1}(x) = \dfrac{2x+3}{5x-2}}$

46. What makes a function one-to-one?

When for each x-value there is only one y-value.

47. Are the following functions inverses? (Need to show $f(g(x)) = x$ and $g(f(x)) = x$)

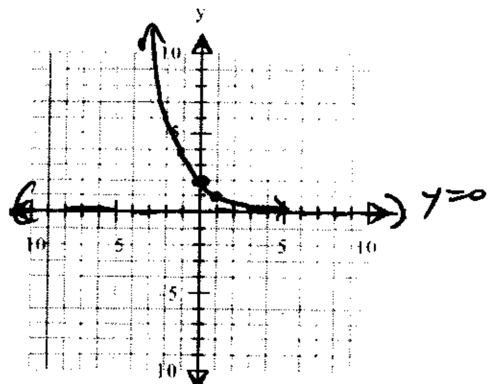
$f(x)=3x-6$ $\qquad g(x)=\frac{1}{3}x+2$

$f(g(x)) = f(\frac{1}{3}x+2) = 3(\frac{1}{3}x+2) - 6$
$= x+6-6 = x$ ✓

$g(f(x)) = g(3x-6) = \frac{1}{3}(3x-6)+2$
$= x-2+2 = x$ ✓

yes, inverses

48. Graph $f(x)=\left(\frac{1}{2}\right)^{x-1}$ Identify the transformations, intercepts, asymptotes, domain and range. Use 3 key points.




Parent function: $y = \frac{1}{2}^x$

parent table:

| x | y |
|---|---|
| -1 | 2 |
| 0 | 1 |
| 1 | 1/2 |
| 2 | 1/4 |

Horiz. Asympt: $y=0$

Transformations: Shift right 1

| x+1 | y |
|---|---|
| 0 | 2 |
| 1 | 1 |
| 2 | 1/2 |
| 3 | 1/4 |

key points from table

Horiz. Asympt. $y=0$

Intercepts: $(0,2) \to$ y-int ; no x-int.

Domain: $(-\infty,\infty)$

Range: $(0,\infty)$

49. Find the domain of $f(x)=\ln(10-x)$. Show work!

$10 - x > 0$

$-x > -10$

$\boxed{x < 10 \text{ or } (-\infty, 10)}$

50. Graph $f(x)=\log_2 x+1$. Identify the transformations, intercepts, asymptotes, domain and range. Use 3 key points.

Parent graph: $y=\log_2 X$

Parent table:

| X | Y |
|---|---|
| $\frac{1}{2}$ | -1 |
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |

V.A: $X=0$

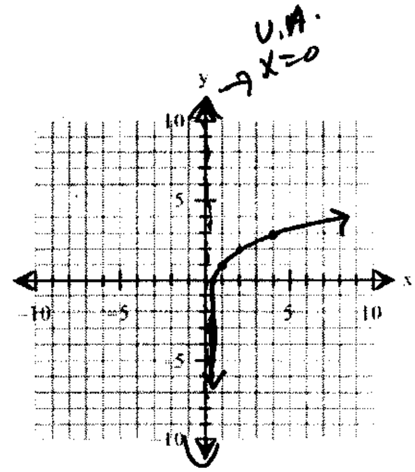
Transformations: up 1

Intercepts: $(\frac{1}{2},0)$ → x-int. no y-int.

Asympt.: $X=0$

Domain: $X>0$, or $(0,\infty)$

Range: $(-\infty,\infty)$




$y=\log_2 X+1$

| X | y+1 |
|---|---|
| $\frac{1}{2}$ | 0 |
| 1 | 1 |
| 2 | 2 |
| 4 | 3 |

key points from table

51. Find the amount which results from the following investment. \$10,000 invested at 8% compounded quarterly after a period of 5 years.

$$A=P\cdot\left(1+\frac{r}{n}\right)^{nt}$$

$P=10{,}000$
$r=.08,\ n=4,\ t=5$

$$A=10{,}000\left(1+\frac{.08}{4}\right)^{4\cdot 5}=\boxed{\$14{,}859.47}$$

52. The formula for a small bacteria population is $P(t)=400e^{.23t}$ After how many years will the population reach 2000?

$$\frac{2000}{400}=\frac{400e^{.23t}}{400}$$

$$5=e^{.23t}$$

$$\ln 5=.23t$$

$$\frac{\ln 5}{.23}=t$$

$t=6.99$ Approx. 7 years.

53. The half-life of Wellsonium is 630 years. If 50 grams are present now how much will be present in 800 years? Round to the nearest hundredth.

$A = Pe^{rt}$, Find r first, then find t using the r.

$\frac{1}{2} = e^{r(630)}$

$\ln(\frac{1}{2}) = 630r$

$\frac{\ln(.5)}{630} = r$

$-.0011 = r$ (round to 4-decimals)

$A = 50 e^{(-.0011 \cdot 800)}$

$A = \boxed{20.74 \text{ grams}}$

**Solve each equation using substitution. Show work. Show answer as exact and as a decimal rounded to the nearest four decimal places.**

54. $e^{2x} - 2e^x - 3 = 0$

$u = e^x$

$u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

$u = 3 \quad u = -1$

$e^x = 3 \quad \cancel{e^x = -1}$

$\ln 3 = x \approx 1.0986$

55. $3^{2x} + 3^x - 2 = 0$

let $u = 3^x$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2 \quad u = 1$

$\cancel{3^x = -2} \quad 3^x = 1 \quad \boxed{x = 0}$

56. Find $f \circ g(x)$ $\quad f(x) = \sqrt{x+2}$ $\quad g(x) = 2x^2 + 1$

$f(g(x)) = f(2x^2+1) = \sqrt{2x^2+1+2}$

$= \boxed{\sqrt{2x^2+3}}$

57. $f(x) = \frac{x+1}{x-1}$ $\quad g(x) = \frac{1}{x}$ $\quad$ *Find domain of $f \circ g$.*

find domain of g first

$x \neq 0$.

Find domain of f.

$x \neq 1$, so $g(x) \neq 1$, so $\frac{1}{x} \neq 1$

So, $x \neq 1$

Domain: $x \neq 0$ + $x \neq 1$ or $(-\infty, 0) \cup (0,1) \cup (1, \infty)$